MTH 605: Topology I

Practice Assignment III

1. Show that every connected graph has a maximal tree.
2. Show that the Cayley complex $\widetilde{X_G}$ is the universal covering space of X_G.
3. Construct $\widetilde{X_G}$, when $G = \mathbb{Z}_n$.
4. If X is a finite graph and Y is a subgraph homeomorphic to S^1 and containing the basepoint x_0, show that $\pi_1(X, x_0)$ has a basis in which one element is represented by the loop in Y.
5. If F is a finitely generated free group and N is a nontrivial normal subgroup of infinite index, then show (using covering spaces) that N is not finitely generated.
6. Find all connected covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$ up to isomorphism.
7. Find the covering space of $S^1 \vee S^1$ (where $\pi_1(S^1 \vee S^1) = F(a, b)$), corresponding to the normal subgroup generated by a^2, b^2 and $(ab)^4$.
8. Show that the closed orientable surface S_g of genus g cannot retract onto a separating curve, but it does retract onto a nonseparating curve.